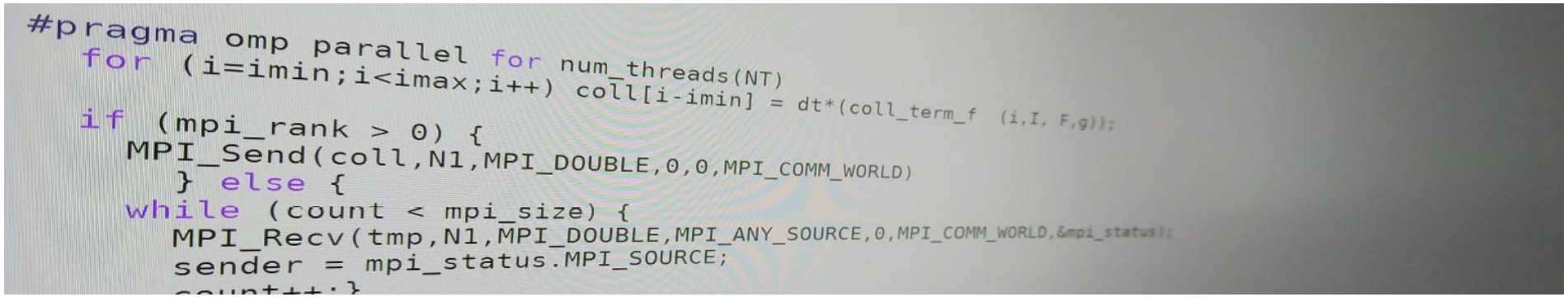


Introduction to parallel programming (for physicists)

FRANÇOIS GÉLIS & GRÉGOIRE MISGUICH, IPhT courses, June 2019.

IPhT, CEA-Saclay
Itzykson room
courses.ipht.fr

1. Introduction & hardware aspects (FG)
2. A few words about Maple & Mathematica
3. Linear algebra libraries
4. Fast Fourier transform
5. Python Multiprocessing
6. OpenMP
7. MPI (FG)
8. MPI+OpenMP (FG)

These slides (GM)

Numerical Linear algebra

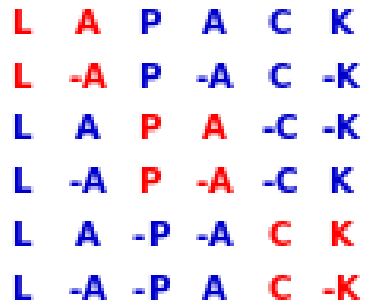
Here: a few simple examples
showing how to call some parallel
linear algebra libraries
in numerical calculations

(numerical) Linear algebra

- Basic Linear Algebra Subroutines: **BLAS**
  - vector op. (=level 1)
  - matrix-vector (=level 2)
  - matrix-matrix mult. & triangular inversion (=level 3)
  - Many implementations but standardized interface
  - Discussed here: Intel MKL & OpenBlas (multi-threaded = parallelized for shared-memory architectures)

- More advanced operations Linear Algebra Package: **LAPACK**

  ('90, Fortran 77)
  - Call the BLAS routines
  - Matrix diagonalization, linear systems and eigenvalue problems
  - Matrix decompositions: LU, QR, SVD, Cholesky
  - Many implementations



Used in most scientific softwares & libraries
(Python/Numpy, Maple, Mathematica, Matlab, …)

A few other useful libs

… for BIG matrices

- **ARPACK**

=Implicitly Restarted Arnoldi Method (~Lanczos for Hermitian cases)

Large scale eigenvalue problems. For (usually **sparse)** nxn matrices with n which can be as large as 10^8, or even more !
Ex: iterative algo. to find the largest eigenvalue, without storing the matrix M (just provide v→Mv).

Can be used from Python/SciPy

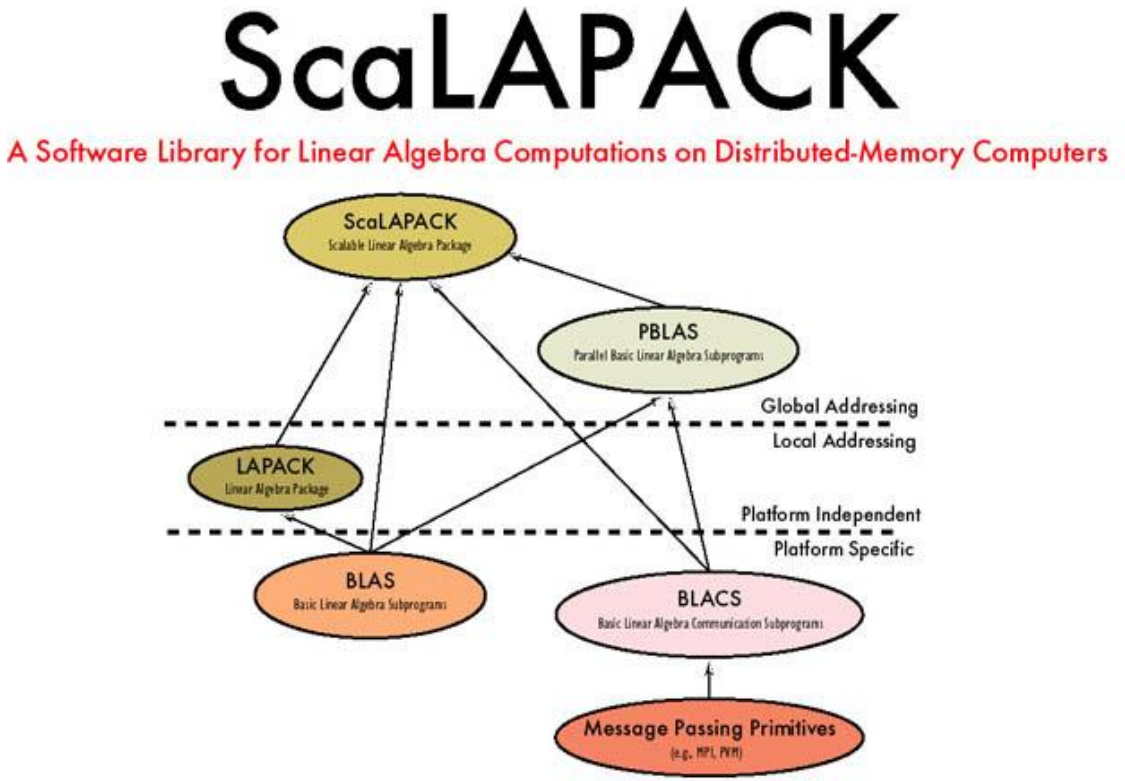
- **PARPACK**

= parallel version of ARPACK

for distributed memory architectures (the matrices are stored over several nodes)

- **ScaLAPACK**

Parallel version of LAPACK for for distributed memory architectures



Two multi-threaded implementations of BLAS & Lapack

OpenBLAS

- Open source License (BSD)
- Based on GotoBLAS2
  [Created by GAZUSHIGE GOTO, Texas Adv. Computing Center, Univ. of Texas]
- Can be used from Fortran, C, C++, Python/Numpy, …

Intel's implementation (=part of the MKL lib.)

- Commercial license
- Included in *Intel® Parallel Studio XE* (compilers, libraries, …), which is free for students
- Included in *Intel® Performance Libraries* (librairies only, without compiler), free for every one.
- Installed in most/many computing centers / intel-based clusters
- Included in `intel-python`, which is free for *all* users
- Can be used from Fortran, C, C++ or Python/Numpy

Lapack/Intel-MKL

Code examples in C or FORTRAN:
https://software.intel.com/sites/products/documentation/doclib/mkl_sa/11/mkl_lapack_examples/

Intel® Math Kernel Library LAPACK Examples

This document provides code examples for LAPACK (Linear Algebra PACKage) routines that solve problems in the following fields:

Linear Equations

Examples for several LAPACK routines that solve systems of linear equations.

Linear Least Squares Problems

Examples for some of the LAPACK routines that find solutions to linear least squares problems.

Symmetric Eigenproblems

Symmetric Eigenproblems has examples for LAPACK routines that compute eigenvalues and eigenvectors of real symmetric and complex Hermitian matrices.

Nonsymmetric Eigenproblems

Nonsymmetic Eigenproblems provides examples for *?geev*, one of several LAPACK routines that compute eigenvalues and eigenvectors of general matrices.

Singular Value Decomposition

Examples for LAPACK routines that compute the singular value decomposition of a general rectangular matrix.

Lapack-MKL example in Fortran

DGELSD/from Intel's website (part 1/3)

```
*  The routine computes the minimum-norm solution to a real linear least
*  squares problem: minimize ||b - A*x|| using the singular value
*  decomposition (SVD) of A. A is an m-by-n matrix which may be
*  rank-deficient.
*
*  Several right hand side vectors b and solution vectors x can be
*  handled
*  in a single call; they are stored as the columns of the m-by-nrhs
*  right
*  hand side matrix B and the n-by-nrhs solution matrix X.
*
*  The effective rank of A is determined
*  singular
*  values which are less than rcond times
*
*  Example Program Results.
*  ========================
*
* DGELSD Example Program Results
*
* Minimum norm solution
*  -0.69  -0.24   0.06
*  -0.80  -0.08   0.21
*   0.38   0.12  -0.65
*   0.29  -0.24   0.42
*   0.29   0.35  -0.30
*
(...)
```

Here minimize $\left\|\vec{b} - A\vec{x}\right\|_2$ with A a rectangular matrix and $\vec{b}$ a vector.

Naming convention of the LAPACK routines : *XYYZZZ*

X :	S	REAL
	D	DOUBLE PRECISION
	C	COMPLEX
	Z	COMPLEX*16 or DOUBLE COMPLEX
YY:	BD	bidiagonal
	DI	diagonal
	GB	general band
	GE	general
	GG	general matrices, generalized problem (i.e., a pair of general matrices)
	GT	general tridiagonal
	(...)	

ZZZ: Type of computation. Here LSD stands for minimum norm solution to a linear least squares problem using the singular value decomposition of A and a divide and conquer method

Lapack-MKL example in Fortran

(part 2/3)

```
*      .. Parameters ..
       INTEGER            M, N, NRHS
       PARAMETER          ( M = 4, N = 5, NRHS = 3 )
       INTEGER            LDA, LDB
       PARAMETER          ( LDA = M, LDB = N )
       INTEGER            LWMAX
       PARAMETER          ( LWMAX = 1000 )
*
*      .. Local Scalars ..
       INTEGER            INFO, LWORK, RANK
       DOUBLE PRECISION   RCOND
*
*      .. Local Arrays ..
*      IWORK dimension should be at least
*      3*MIN(M,N)*NLVL + 11*MIN(M,N),
* Where
* NLVL = MAX( 0,INT(LOG_2( MIN(M,N)/(SMLSIZ+1)))+1)
* and SMLSIZ = 25
INTEGER            IWORK( 3*M*0+11*M )
DOUBLE PRECISION   A( LDA, N ), B( LDB, NRHS ),S( M ),
                   WORK( LWMAX )
      DATA               A/
     $  0.12,-6.91,-3.33, 3.97,
     $ -8.19, 2.22,-8.94, 3.33,
     $  7.69,-5.12,-6.72,-2.74,
     $ -2.26,-9.08,-4.40,-7.92,
     $ -4.71, 9.96,-9.98,-3.20
     $                   /
      DATA               B/
     $  7.30, 1.33, 2.68,-9.62, 0.00,
     $  0.47, 6.58,-1.71,-0.79, 0.00,
     $ -6.28,-3.42, 3.46, 0.41, 0.00
     $                   /
*
*      .. External Subroutines ..
       EXTERNAL           DGELSD
       EXTERNAL           PRINT_MATRIX
*
*      .. Intrinsic Functions ..
       INTRINSIC          INT, MIN
```

Lapack routines require you to provide some memory space to work (in the form of array(s)). Here: `WORK`, double precision array of size `LWORK` (see next slide).

`M`, `N`: size of A
`NRHS`: number of vectors $\vec{b}$ for which the problem must me solved.

Lapack-MKL example in Fortran (part 3/3)

```
*     .. Executable Statements ..
WRITE(*,*)'DGELSD Example Program
Results'
*     Negative RCOND means using
default (machine precision) value
RCOND = -1.0
*
*     Query the optimal workspace.
*
LWORK = -1
CALL DGELSD( M, N, NRHS, A, LDA, B,
LDB, S, RCOND, RANK, WORK, LWORK,
IWORK, INFO )
LWORK = MIN( LWMAX, INT( WORK( 1 )))
*
*     Solve the equations A*X = B.
*
CALL DGELSD( M, N, NRHS, A, LDA, B,
LDB, S, RCOND, RANK, WORK, LWORK,
IWORK, INFO )
*
*     Check for convergence.
*
IF( INFO.GT.0 ) THEN
WRITE(*,*)'The algorithm computing
SVD failed to converge;'
WRITE(*,*)'the least squares solution
could not be computed.'
STOP
END IF
*
*     Print minimum norm solution.
*
CALL PRINT_MATRIX( 'Minimum norm
solution', N, NRHS, B, LDB )
*
*     Print effective rank.
*
WRITE(*,'(/A,I6)')' Effective rank =
', RANK
*
*     Print singular values.
*
CALL PRINT_MATRIX( 'Singular values',
1, M, S, 1 )
STOP
END
*
*     End of DGELSD Example.
```

First call to `DGELSD` with `LWORK=-1` → LAPACK returns the optimal size `LWORK` of the workspace array `WORK`.

Actual calculation

Compilation and output :

```
$ ifort -mkl DGELSD_example.f
$ ./a.out
 DGELSD Example Program
Results

 Minimum norm solution
  -0.69   -0.24    0.06
  -0.80   -0.08    0.21
   0.38    0.12   -0.65
   0.29   -0.24    0.42
   0.29    0.35   -0.30

 Effective rank =      4

 Singular values
  18.66   15.99   10.01    8.51
```

Lapack/Intel-MKL

zheevd example in C

```c
#include <stdlib.h>
#include <stdio.h>
#include <mkl.h>
// C-wrapper to the Fortran Lapack lib. :
#include <mkl_lapacke.h>
int main() {
const int n=2000;

printf("Matrix size=%i\n",N);
printf("Number of threads=%i\n",
mkl_get_max_threads());

MKL_INT N = n, LDA = n, info,i,j;
double w[N];
MKL_Complex16* a;
a=malloc(N*LDA*sizeof(MKL_Complex16));


srand(999); /* dense random matrix */
for (i=0; i < N; i++ )
   for(j = 0; j<N; j++ )
a[i*LDA+j].real=rand(),a[i*LDA+j].imag=rand(
);
```

prints the num. of threads

```c
  // LAPACKE_zheevd: computes all
  // eigenvalues and eigenvectors of a
  // complex Hermitian matrix A using divide
  // and conquer algorithm

info = LAPACKE_zheevd( LAPACK_ROW_MAJOR,
'V', 'L', N, a, LDA, w );

  /* Check for convergence */
  if( info > 0 ) {
    printf( "The algorithm failed to compute
eigenvalues.\n" );
    exit( 1 );
  }
  /* Print the extreme eigenvalues */
printf("Smallest eigen value=%6.2f\n",w[0]);
printf("Largest value=%6.2f\n\n",w[N-1]);
}
```

w: array containing the eigen. vals.

Call to Lapack.
This version takes care of the workspace memory management (contrary to the Fortran version)

OpenBLAS/LAPACK

zheevd example in C

```c
#include <stdlib.h>
#include <stdio.h>
#include <omp.h>
// C-wrapper to the Fortran Lapack lib.
#include <lapacke.h>
int main() {
const int N=2000;

printf("Matrix size=%i\n",N);
printf("Number of threads=%i\n",
omp_get_max_threads());

int LDA=N, info,i,j;
double w[N];
lapack_complex_double* a;
a=malloc(N*LDA*sizeof(lapack_complex_double)
);

srand(999); /* Dense random matrix */
for (i=0; i < N; i++ )
   for(j = 0; j<N; j++ )
a[i*LDA+j]=lapack_make_complex_double(rand()
,rand());

  // LAPACKE_zheevd: computes all
  // eigenvalues and eigenvectors of a
  // complex Hermitian matrix A using divide
  // and conquer algorithm

info = LAPACKE_zheevd( LAPACK_ROW_MAJOR,
'V', 'L', N, a, LDA, w );

  /* Check for convergence */
  if( info > 0 ) {
    printf( "The algorithm failed to compute
eigenvalues.\n" );
    exit( 1 );
  }
  /* Print the extreme eigenvalues */
printf("Smallest eigen value=%6.2f\n",w[0]);
printf("Largest value=%6.2f\n\n",w[N-1]);
}
```

OpenBLAS/CBLAS

dgemm example in C

```c
#include <stdio.h>
#include <stdlib.h>
#include <cblas.h>
int main() {
int N=10000,N2,i,j;
N2=N*N;
//Memory allocation for the arrays:
double *A, *B, *C;
A = (double *)malloc( N2*sizeof( double ) );
B = (double *)malloc( N2*sizeof( double ) );
C = (double *)malloc( N2*sizeof( double ) );

for (i = 0; i < (N2); i++)
        A[i] = (double)(i+1),
        B[i] = (double)(-i-1),
        C[i] = 0.0;
printf ("Computing matrix product using OpenBLAS dgemm
function via CBLAS interface ...\n");
cblas_dgemm(CblasRowMajor, CblasNoTrans, CblasNoTrans,
                N, N, N, 1.0, A, N, B, N, 1.0, C, N);
printf ("done.\n\n");
return 0;
}
```

Matrix-matrix multiplcation (BLAS) of double precision general matrices

Specify the « order » of the matrix elements in memory:

A[i][j]=A[j+LDA*i] row-major
(C/C++ order)

A[i][j]=A[i+LDA*j] column-major
(=Fortran order)

Compilation of the OpenBLAS examples – use of `make` & `makefile`

```
#specify below the path to the OpenBLAS library files (like libopenblas.a and cblas.h)
OPEN_BLAS_LIB= ../OpenBLAS
#specify below the path to the lapacke.h header file
LAPACKE_INC=   ../OpenBLAS/lapack-netlib/LAPACKE/include

dgemm_example_mini.exe: dgemm_example_mini.c
	gcc $< -o $@ -L $(OPEN_BLAS_LIB)-I $(OPEN_BLAS_LIB) -lopenblas -pthread
zheevd_example.exe: zheevd_example.c
	gcc $< -o $@ -I $(LAPACKE_INC) -L $(OPEN_BLAS_LIB) -lopenblas -fopenmp -lgfortran
clean:
	\rm *.exe
all: dgemm_example.exe zheevd_example.exe
```

`makefile`
$< : 1st pre-requiste (usually the source file)
$@ : target (executable name)

`-j` option: use several threads/cores to compile multiple files in parallel

To compile: `make dgemm_example.exe` or `make all` or `make -j 2 all`

Check parallelism (1)

```
$ make  dgemm_example.exe
gcc dgemm_example.c -o dgemm_example.exe -L ../OpenBLAS -I ../OpenBLAS
-lopenblas -fopenmp -lrt

$ export OMP_NUM_THREADS=1; ./dgemm_example.exe 2000
Initializing the matrices ... done.
Computing matrix product using OpenBLAS dgemm function via CBLAS
interface... done.
Elaspe time (s): 0.719543
22.2308 GFlops

$ export OMP_NUM_THREADS=10; ./dgemm_example.exe 2000
Initializing the matrices ... done.
Computing matrix product using OpenBLAS dgemm function via CBLAS
interface... done.
Elaspe time (s): 0.0814542
196.38 GFlops
```

Close to the peak power of the CPU
(here Xeon E5-2630 v2 @ 2.60GHz / 15360 KB Cache).
Check with **`cat /proc/cpuinfo`**)

Check parallelism (2)

Environment variable to specify the # of threads (if not specified in the code)

```
$ export OMP_NUM_THREADS=10; ./dgemm_example.exe 10000
Initializing the matrices ... done.
Computing matrix product using OpenBLAS dgemm function via CBLAS interface... done.
Elaspe time (s): 9.45623
211.49 GFlops
```

1 process with ~10 threads running

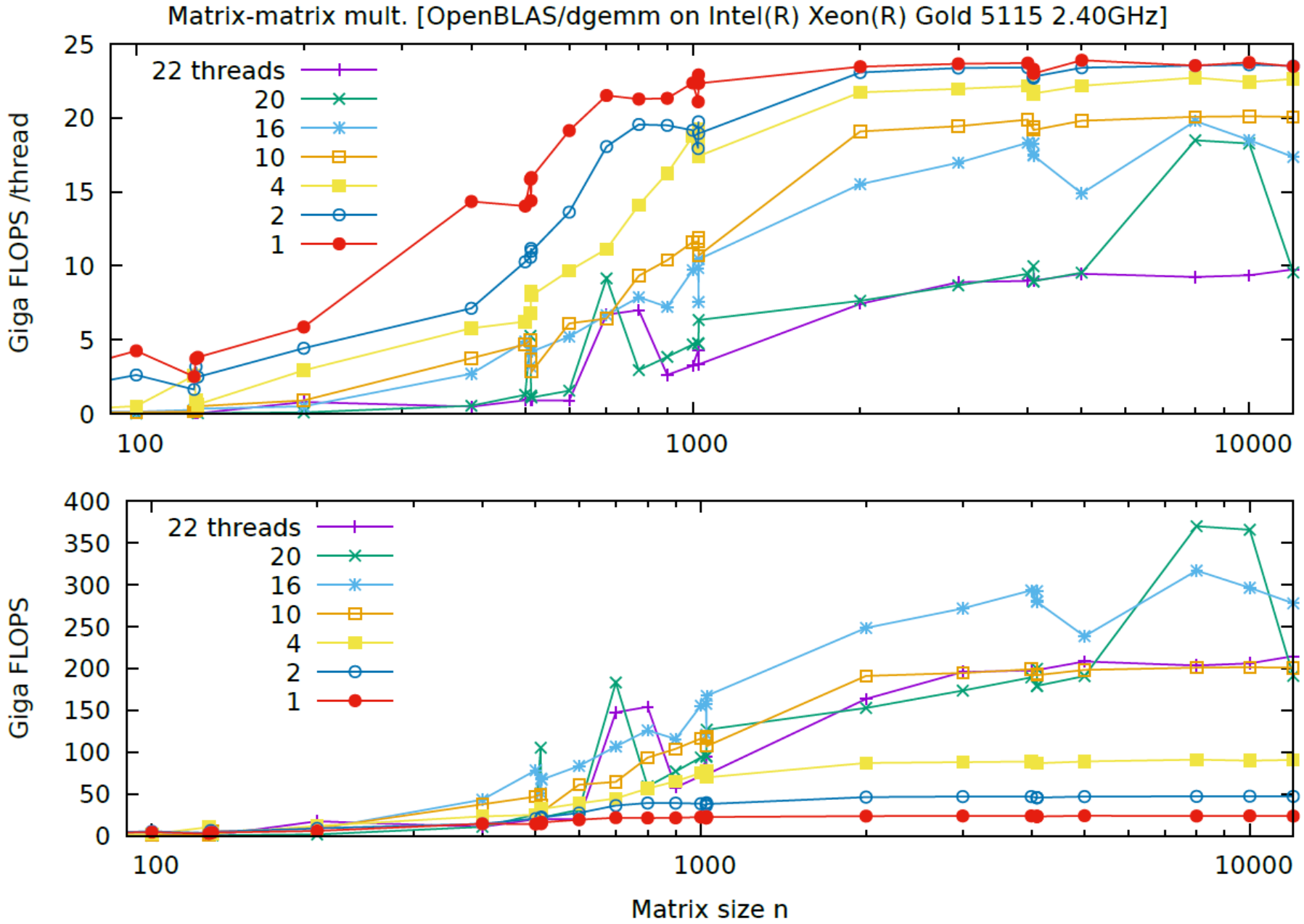
Top

misguich@totoro:~

```
top - 11:20:56 up 5 days, 19:04,  7 users,  load average: 3.56, 1.75, 0.74
Tasks: 547 total,   2 running, 545 sleeping,   0 stopped,   0 zombie
Cpu(s): 41.6%us,  0.1%sy,  0.0%ni, 58.2%id,  0.1%wa,  0.0%hi,  0.0%si,  0.0%st
Mem:  65876860k total,  9536948k used, 56339912k free,  1465352k buffers
Swap: 65535996k total,        0k used, 65535996k free,  4382076k cached

  PID USER      PR  NI  VIRT  RES  SHR S %CPU %MEM    TIME+  COMMAND
75562 misguich  20   0 2799m 2.3g  616 R 999.0  3.6   0:53.36 dgemm_example.e
 3559 nrpe      20   0 43596 1432 1012 S  0.3  0.0   0:06.47 nrpe
 3694 root      20   0 62852  40m 1060 S  0.3  0.1   3:53.52 pbs_server
75387 misguich  20   0 17508 1664  972 R  0.3  0.0   0:00.53 top
    1 root      20   0 21456 1624 1300 S  0.0  0.0   0:03.70 init
```

OpenBLAS performance

dgemm (from C)



Anatomy of High-Performance Matrix Multiplication

KAZUSHIGE GOTO
The University of Texas at Austin
and
ROBERT A. VAN DE GEIJN
The University of Texas at Austin

High-performance BLAS

Now *OpenBLAS*

We present the basic principles which underlie the high-performance implementation of the matrix-matrix multiplication that is part of the widely used GotoBLAS library. Design decisions are justified by successively refining a model of architectures with multilevel memories. A simple but effective algorithm for executing this operation results. Implementations on a broad selection of architectures are shown to achieve near-peak performance.

Categories and Subject Descriptors: G.4 [**Mathematical Software**]: —*Efficiency*

General Terms: Algorithms;Performance

Additional Key Words and Phrases: linear algebra, matrix multiplication, basic linear algebra subprogrms

ACM Transactions on Mathematical Software 34(3):Article 12 · May 2008. DOI: 10.1145/1356052.1356053

Armadillo: C++ library for linear algebra & scientific computing

Example of (symmetric) matrix diagonalization

Can call OpenBLAS or MKL

```
#include <iostream>
#include <armadillo>
using namespace arma;
int main() {
      const int N=5000;
      size_t dim=N;
      mat A(dim, dim,
arma::fill::randu);
      vec eigval;
      mat eigvec;
      eig_sym(eigval,eigvec,A);
      cout<<"1st
eigenvalue="<<eigval[0]
      <<"\tLast="<<eigval[dim-
1]<<endl;
return 0;
```

Armadillo: C++ library for linear algebra & scientific computing

Matrix-matrix multiplication

```
#include <iostream>
#include <armadillo>
using namespace arma;
int main() {
const int N=10000;
size_t dim=N;
mat A(dim, dim, arma::fill::randu);
mat B(dim, dim, arma::fill::randu);
mat C=A*B;
return 0;
}
```

Armadillo: C++ library for linear algebra & scientific computing

compilation

```
ARMA_INC=/usr/local/install/armadillo-9.200.6/include
ARMA_LIB=/usr/local/install/armadillo-9.200.6/lib64/

diag.exe: diag.cpp
	g++ -std=gnu++11 $< -o $@ -I $(ARMA_INC) -L $(ARMA_LIB) -larmadillo

mult.exe: mult.cpp
	g++ -std=gnu++11 $< -o $@ -I $(ARMA_INC) -L $(ARMA_LIB) -larmadillo
```

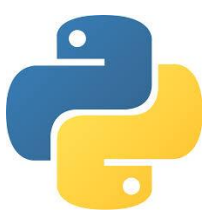
Armadillo: C++ library for linear algebra & scientific computing

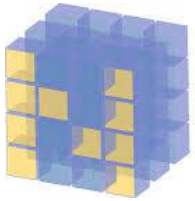
checking what linear algebra library that is actually used

```
$ ldd diag.exe
linux-vdso.so.1 =>  (0x00007ffeddd7c000)
libarmadillo.so.9 => /usr/local/install/armadillo-9.200.6/lib64/libarmadillo.so.9 (0x00007f2c30549000)
libstdc++.so.6 => /usr/local/install/gcc-4.8.0/lib64/libstdc++.so.6 (0x00007f2c3023f000)
libm.so.6 => /lib64/libm.so.6 (0x000000338f200000)
libgcc_s.so.1 => /usr/local/install/gcc-4.8.0/lib64/libgcc_s.so.1 (0x00007f2c30010000)
libc.so.6 => /lib64/libc.so.6 (0x000000338e200000)
libmkl_rt.so => /opt/intel/composer_xe_2013.5.192/mkl/lib/intel64/libmkl_rt.so
(0x00007f2c2fb02000)
libhdf5.so.6 => /usr/lib64/libhdf5.so.6 (0x0000003390a00000)
libz.so.1 => /lib64/libz.so.1 (0x000000338ee00000)
/lib64/ld-linux-x86-64.so.2 (0x000000338de00000)
libdl.so.2 => /lib64/libdl.so.2 (0x000000338ea00000)
```

MKL used here

BLAS & Lapack from Python/Numpy





L A P A C K
L -A P -A C -K
L A P A -C -K
L -A P -A -C K
L A -P -A C K
L -A -P A C -K

Python → Numpy → Linalg → LAPACK → BLAS

Different possible implementations

- standard BLAS/LAPACK (Netlib)
- ATLAS (Automatically Tuned Linear Algebra Software)
- OpenBLAS
- MKL
- …

Multi-threaded /parallel

Python/Numpy linalg

Check the version of Lapack & BLAS numpy is linked to:

```
>>> import numpy as np
>>> np.__config__.show()
```

```
Python 2.7.5 (default, Mar 20 2015, 15:33:03)
[GCC 4.4.7 20120313 (Red Hat 4.4.7-11)] on linux2
Type "help", "copyright", "credits" or "license"
for more information.
>>> import numpy as np
>>> np.__config__.show()
lapack_opt_info:
    libraries = ['openblas', 'openblas']
    library_dirs = ['/usr/local/lib']
    define_macros = [('HAVE_CBLAS', None)]
    language = c
blas_opt_info:
    libraries = ['openblas', 'openblas']
    library_dirs = ['/usr/local/lib']
    define_macros = [('HAVE_CBLAS', None)]
    language = c
openblas_info:
    libraries = ['openblas', 'openblas']
    library_dirs = ['/usr/local/lib']
    define_macros = [('HAVE_CBLAS', None)]
    language = c
(. . .)
```

Python/Numpy linalg

What if my numpy version is *not* linked to a parallel linear algebra lib. ?

- Install OpenBLAS:

```
> sudo apt-get install libopenblas-dev
```

(this will hopefully replace the previous BLAS lib. by OpenBLAS in Numpy)

- Or the Intel Python distribution

https://software.intel.com/en-us/articles/installing-intel-free-libs-and-python-apt-repo

(…)

```
> sudo apt-get install intelpython3
```

The corresponding numpy will be using the MKL lib.

Python/Numpy linalg

Matrix diagonalization example

```
import numpy as np
import numpy.random as npr
import time
npr.seed(2019)
n=3000
A = npr.randn(n,n)
t = time.time()
v = np.linalg.eigvals(A)
td = time.time() - t
print(" Time=%0.4f s" % (td))
```

Specify the #of threads using an environnent variable (bash shell):

```
> export OMP_NUM_THREADS=4
```

Performance *vs* #of cores & matrix dim.

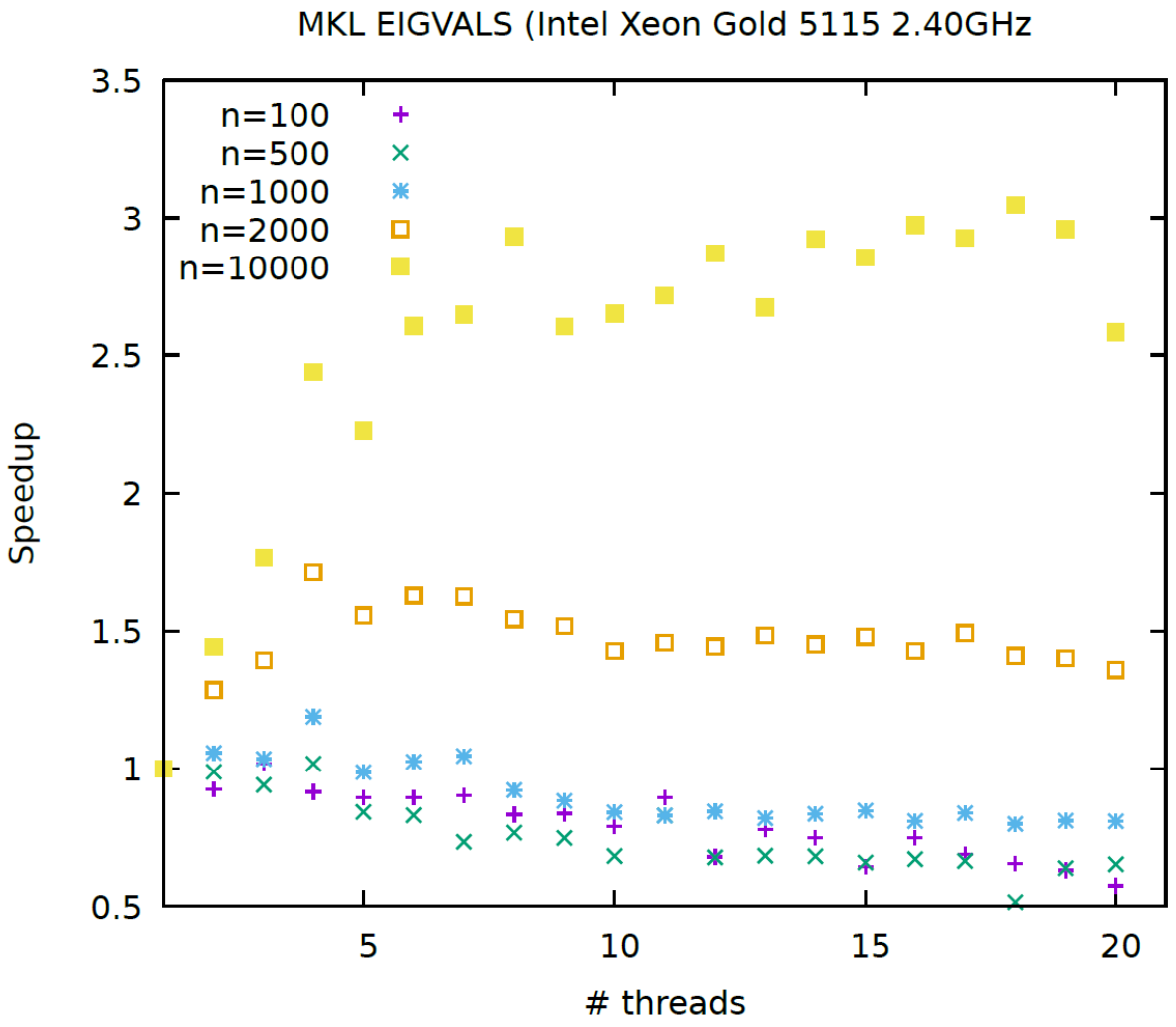
Matrix diagonalization

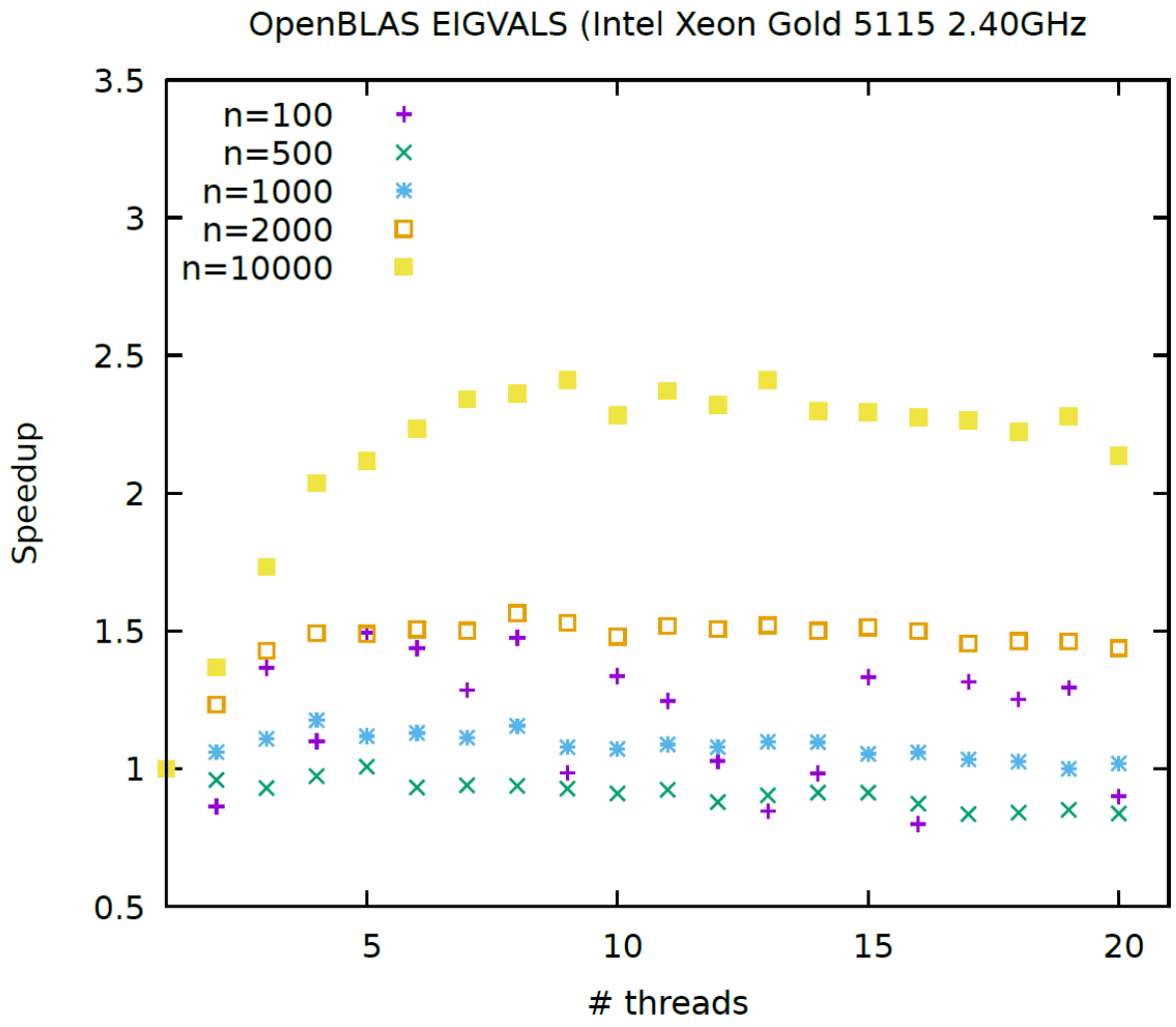
```
import numpy as np
import numpy.random as npr
import time

npr.seed(2019)

sizes=[10,20,100,200,300,400,500,600,700,800,900
,1000,2000,4000,8000,10000,12000]

with open("Eigvals.dat",'w') as my_file:
    my_file.write("#n\ttime\n")
    for n in sizes:
        A = npr.randn(n,n)
        t = time.time()
        v=np.linalg.eigvals(A)
        td = time.time() - t
        print("Eigenvals of (%d,%d) matrix in
%0.4f s" % (n, n, td))
        my_file.write(str(n)+"\t"+str(td)+"\n")
```

```
Eigenvals of (10,10) matrix in 0.0008 s
Eigenvals of (20,20) matrix in 0.0002 s
Eigenvals of (50,50) matrix in 0.0006 s
Eigenvals of (100,100) matrix in 0.0359 s
Eigenvals of (200,200) matrix in 0.0393 s
Eigenvals of (300,300) matrix in 0.0996 s
Eigenvals of (400,400) matrix in 0.1522 s
Eigenvals of (500,500) matrix in 0.2451 s
Eigenvals of (600,600) matrix in 0.3967 s
Eigenvals of (700,700) matrix in 0.4587 s
Eigenvals of (800,800) matrix in 0.5750 s
Eigenvals of (900,900) matrix in 0.6680 s
Eigenvals of (1000,1000) matrix in 0.7734 s
Eigenvals of (2000,2000) matrix in 2.4721 s
Eigenvals of (3000,3000) matrix in 7.4194 s
Eigenvals of (4000,4000) matrix in 14.2675 s
Eigenvals of (6000,6000) matrix in 39.1304 s
Eigenvals of (8000,8000) matrix in 68.1390 s
Eigenvals of (10000,10000) matrix in 109.9833 s
Eigenvals of (12000,12000) matrix in 195.2025 s
```

Performance *vs* #of cores & matrix dim.

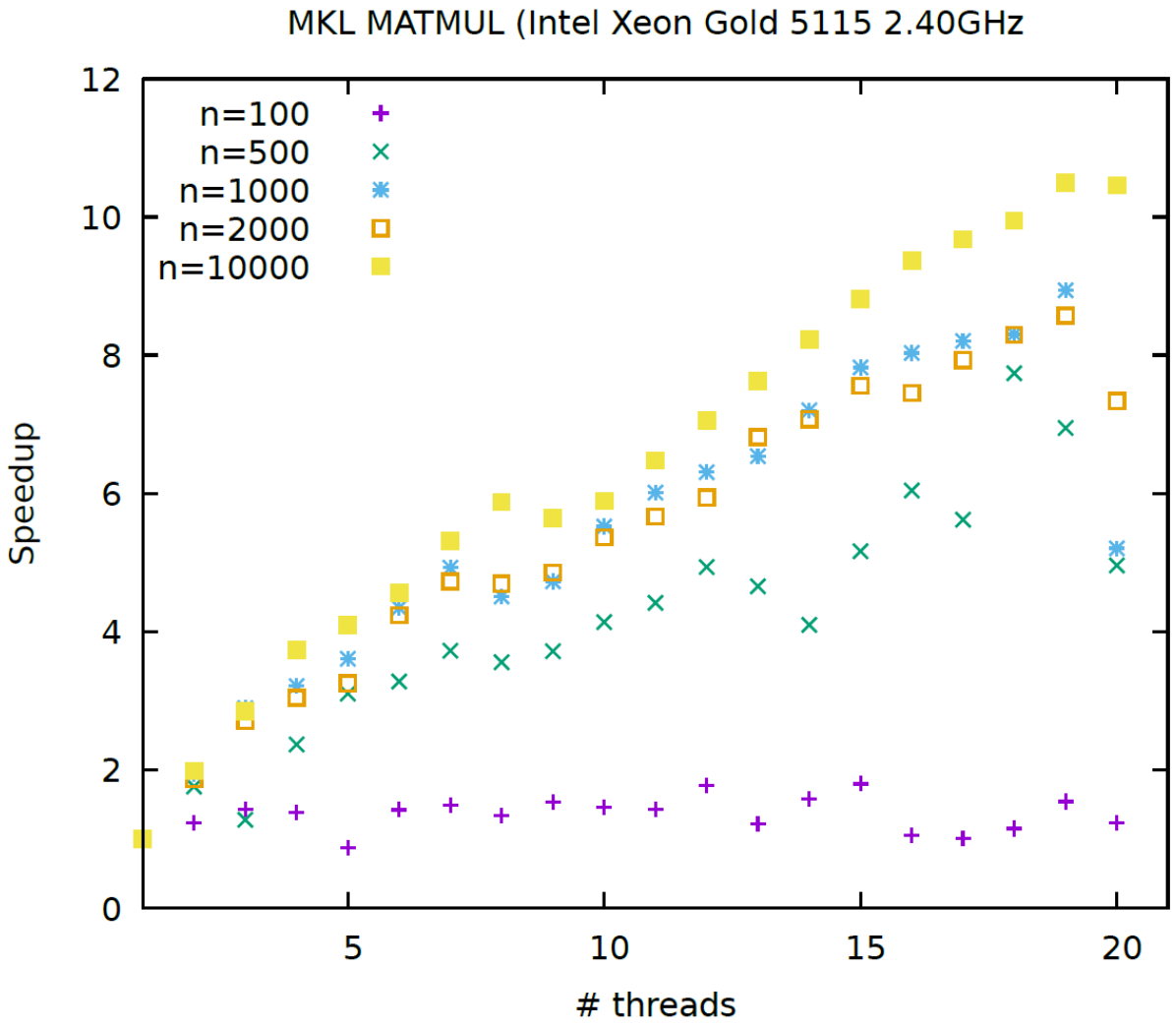
General matrix diagonalization

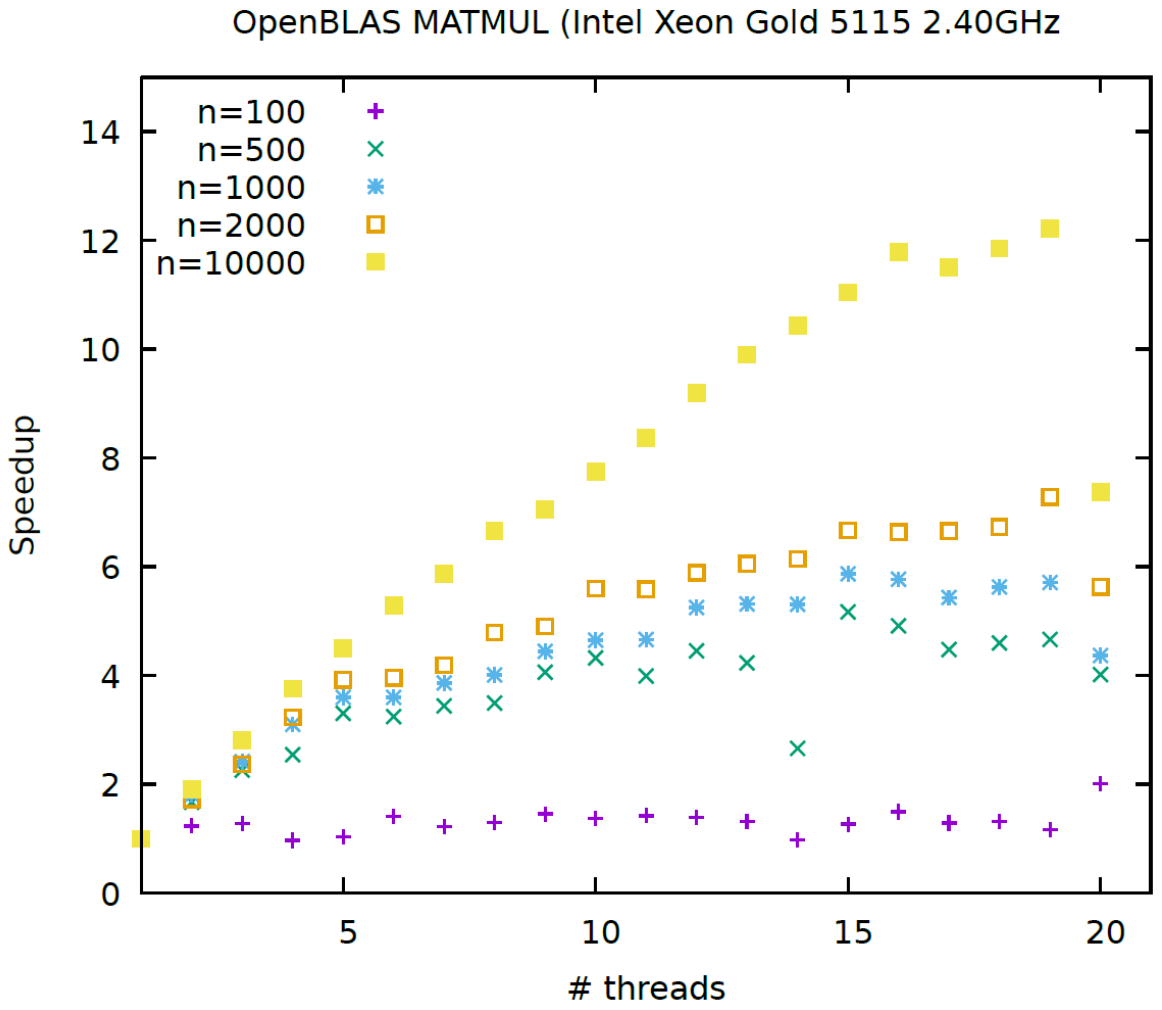




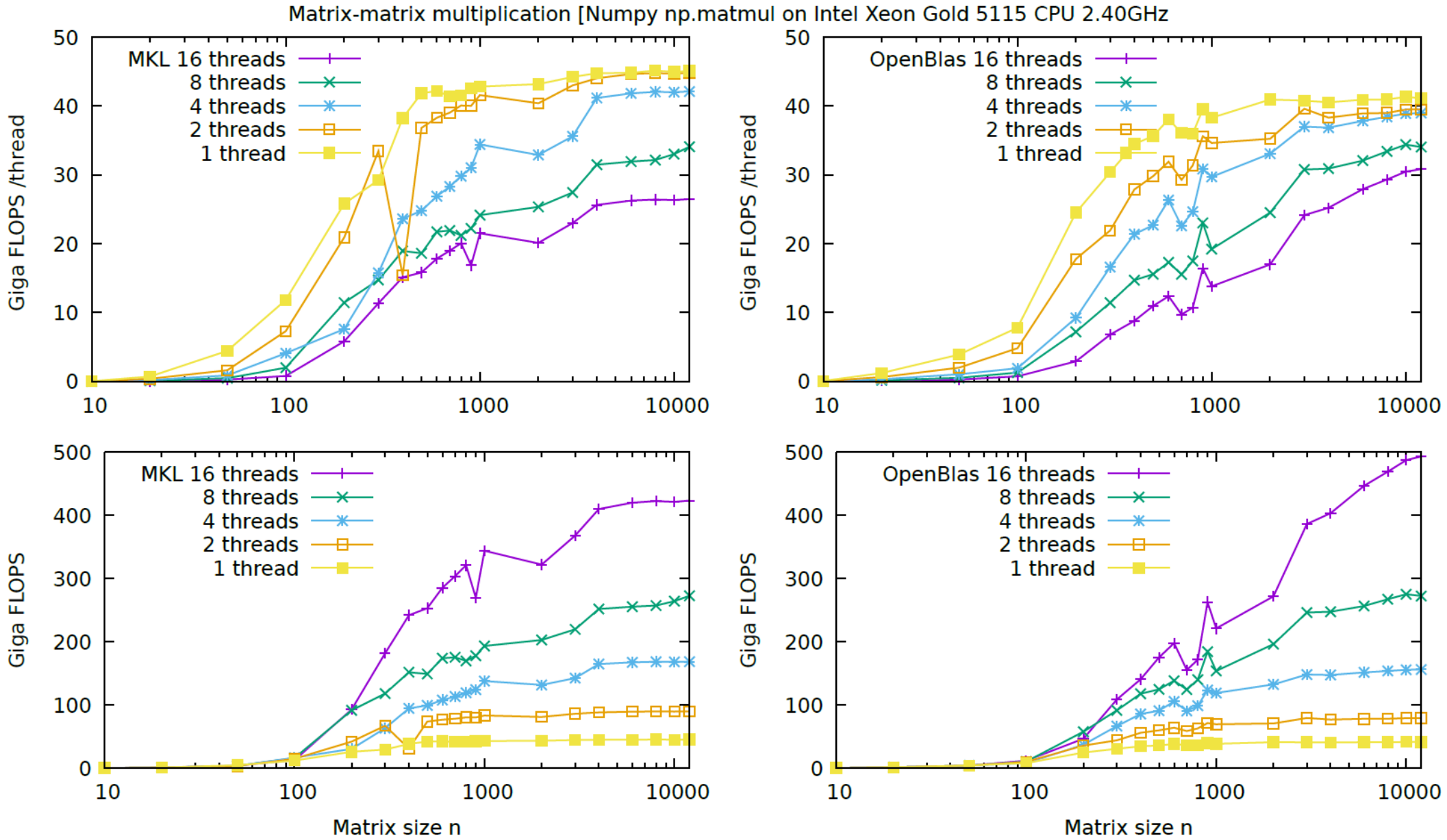
Speedup *vs* #of cores & matrix dim.

Matrix-matrix multiplication



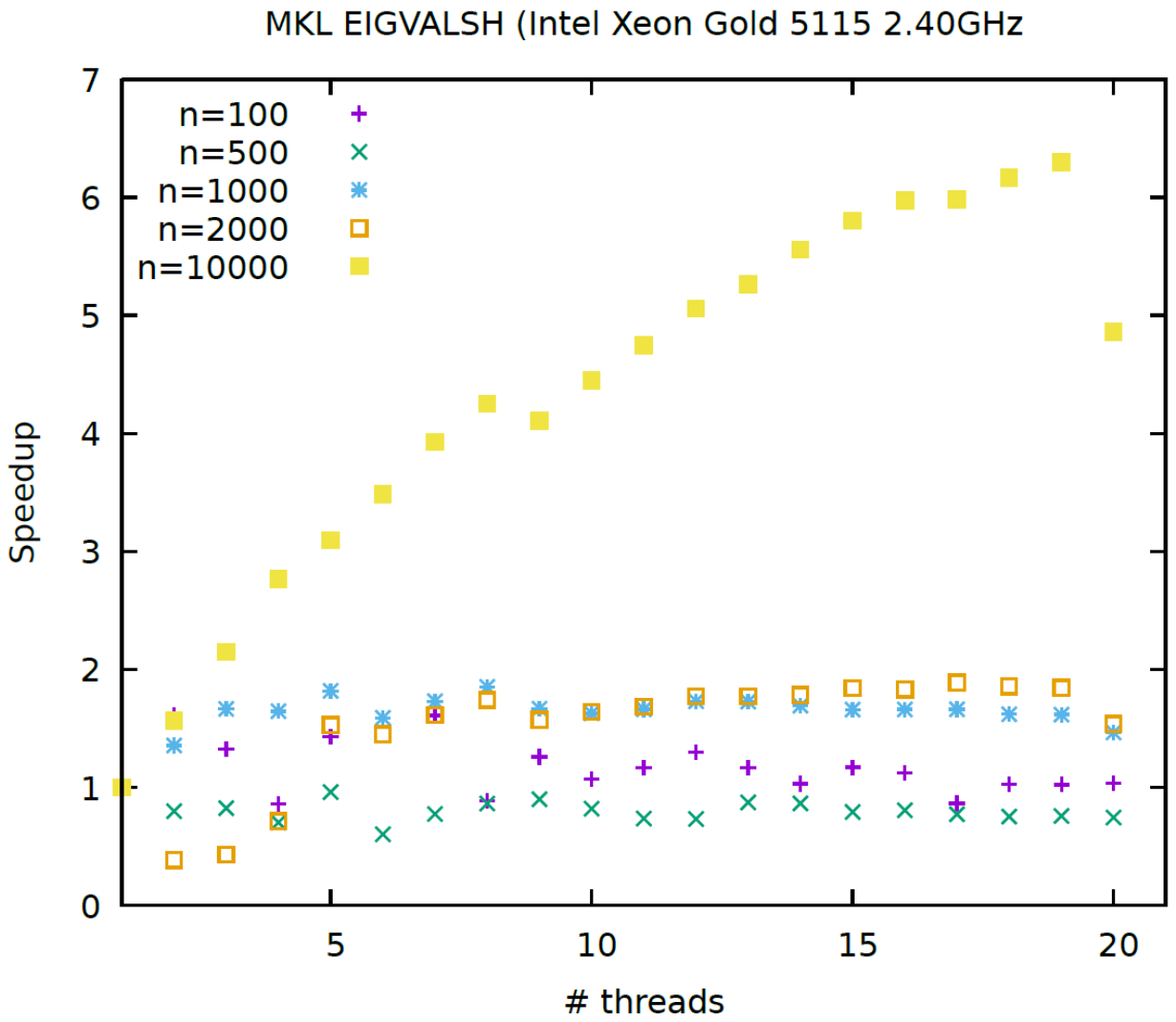


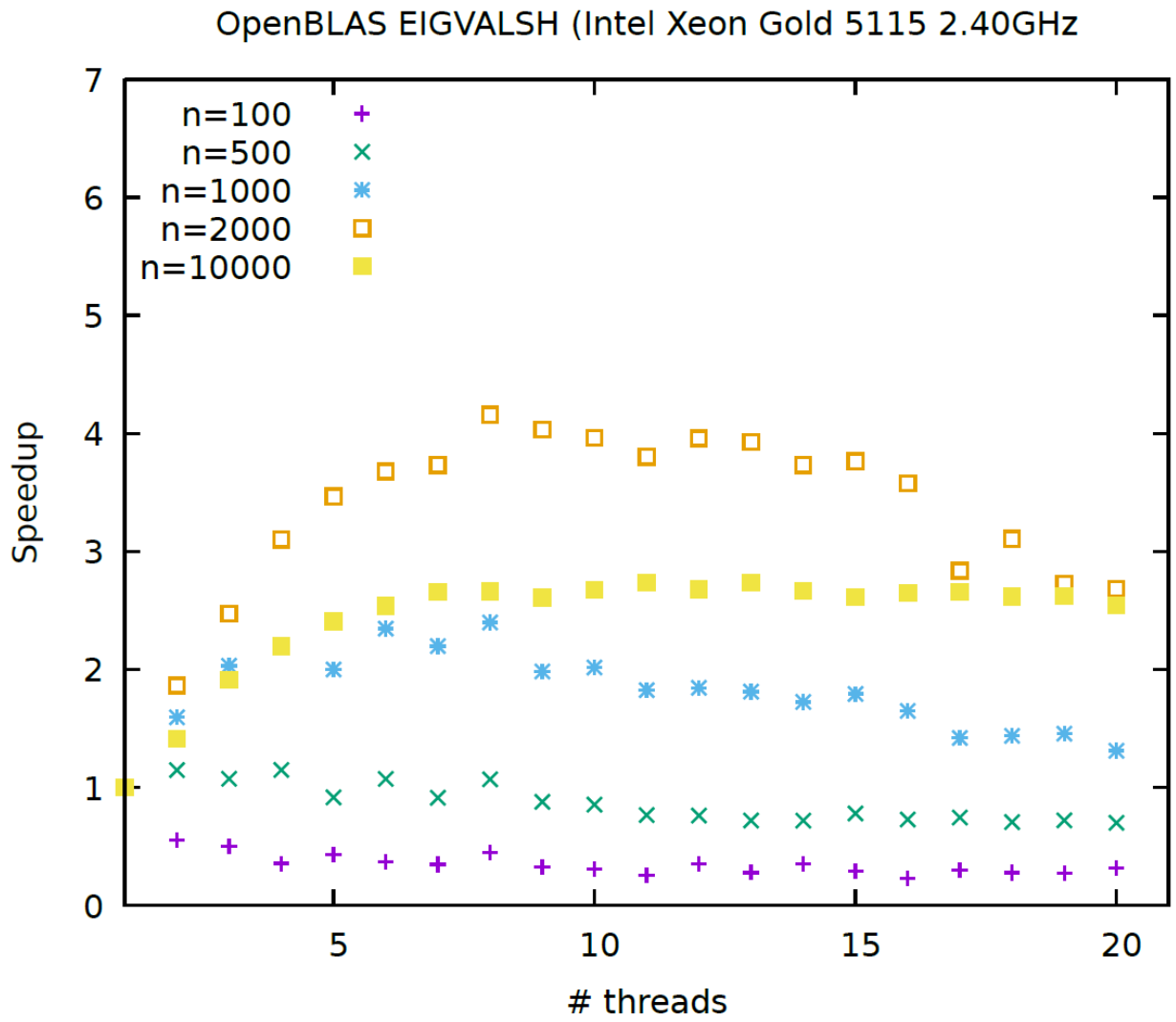
Matrix-matrix multiplication - OpenBLAS & MKL performance



Speedup *vs* #of cores & matrix dim.

Hermitian matrix diagonalization





Speedup *vs* #of cores & matrix dim.

Matrix Singular Value Decomposition